Modelling Agents' Choices in Temporal Linear Logic

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Abstract. Decision-making is a fundamental feature of agent systems. Agents need to respond to requests from other agents, to react to environmental changes, and to prioritize and pursue their goals. Such decisions can have ongoing effects, as the future behavior of an agent may be heavily dependent on choices made earlier. In this paper we investigate a formal framework for modeling the choices of an agent. In particular, we show how the use of a choices calculus based on *temporal linear logic* can be used to capture distribution, temporal and dependency aspects of choices.

1 Introduction

Agents are increasingly becoming accepted as a suitable paradigm for conceptualizing, designing, and implementing the sorts of distributed complex dynamic systems that can be found in a range of domains, such as telecommunications, banking, crisis management, and business transactions [1].

A fundamental theme in agent systems is *decision-making*. Agents have to decide which resources to use, which actions to perform, which goals and commitments to attend to next etc. in order to fulfill their design objectives as well as to respond to other agents in an open and dynamic operating environment. Very often, agents are confronted with choices. Decisions on choices made now may very well affect future achievement of goals or other threads of interactions. In a sense, agents have to make informed and wise decisions on choices. This can be thought of as how to enable agents to act on choices, subject to any possible constraints and with a global consideration to future advantages.

Moreover, in open and dynamic environments, changes from the environment occur frequently and often are unpredictable, which can hinder accomplishment of agents' goals. How agents cope with changes remains an open and challenging problem. On the one hand, agents should be enabled to reason about the current changes and act flexibly. On the other hand, agents should be equipped with a reasoning ability to best predict changes and act accordingly.

These characteristics are desirable for a single agent. However, no agent is an island, and decisions of an agent are not made in isolation, but in the context of decisions made by other agents, as part of interactions between the agents. Thus, the challenging setting here is that in negotiation and other forms of agent interaction, decision making is *distributed*. In particular, key challenges in modeling decision making in agent interaction are:

– *Distribution:* choices are distributed among agents, and changes from the environments affect each agent in different ways. How to capture these choices, their dependencies and the effects of different strategies for their decisions as well as to reason about the global changes at the individual level in agent systems are important.
– *Time:* decision making by agents occurs in time. So do the choices to be made and the changes in the environment. Then it is necessary to deal with them in a time dependent manner.
– *Dependencies:* i.e. capturing that certain decisions depend on other decisions.

The central importance of decision-making in agent systems makes it natural to use logic as a basis for a formal framework for agents. This means that we can model the current state of an agent as a collection of formulas, and the consequences of a particular action on a given state can be explored via standard reasoning methods. In this paper, we explore how to extend this approach to include decisions as well as actions. Hence, for logic-based agents, whose reasoning and decision making is based on a declarative logical formalism, it is important to model the decision making on choices as well as on the environment changes.

This paper tackles the modeling of agent decisions in a way that allows distribution, dependencies, and time of choices to be captured. We discuss specific desirable properties of a formal model of agent choices (section 3) and then present a formal *choice calculus* (section 4). We then consider an application of the choice calculus. Specifically, by ensuring that the choices are made in multiple different formulas consistently, the choice calculus allows us to turn an interaction concerning a goal into multiple concurrent and distributed threads of interaction on its subgoals. This is also based on a mechanism to split a formula Γ which contains A into two formulas, one of which contains A, the other contains the results of "subtracting" A from Γ.

In [2], it was shown how *Temporal Linear Logic* (TLL) can be used to model agent interactions to achieve flexibility, particularly due to its ability to model resources and choices, as well as temporal constraints. This paper can be seen as further developing this line of work to include explicit considerations of the choices of each agent and the strategies of dealing with them.

The remainder of this paper is structured as follows. Section 2 briefly reviews temporal linear logic, and the agent interaction framework. The following two sections motivate and present the choice calculus. Section 5 presents an application of the choice calculus to distributed concurrent problem solving. We then conclude in section 6.

2 Background

2.1 Temporal Linear Logic

Temporal Linear Logic (**TLL**) [3] is the result of introducing temporal logic into linear logic. While linear logic provides advantages to modeling and reasoning about resources, temporal logic addresses the description and reasoning about the changes of truth values of logic expressions over time [4]. Hence, TLL is resource-conscious as well as dealing with time.

In particular, linear logic [5] is well-known for modeling resources as well as updating processes. It has been considered in agent systems to support agent negotiation and planning by means of proof search [6, 7].

In multi-agent systems, utilization of resources and resource production and consumption processes are of fundamental consideration. In such logic as classical or temporal logic, however, a direct mapping of resources onto formulas is troublesome. If we model resources like A as “one dollar” and B as “a chocolate bar”, then $A, A \Rightarrow B$ in classical logic is read as “given one dollar we can get a chocolate bar”. The problem is that A - one dollar - remains afterward. In order to resolve such resource - formula mapping issues, Girard proposed treating formulas as resources and hence they will be used exactly once in derivations.

As a result of such constraint, classical conjunction (and) and disjunction (or) are recast over different uses of contexts - multiplicative as combining and additive as sharing to come up with four connectives. In particular, A $\otimes$ A (*multiplicative conjunction*) means that one has two As at the same time, which is different from $A \wedge A = A$. Hence, $\otimes$ allows a natural expression of proportion. A $\invamp$ B (*multiplicative disjunction*) means that if not A then B or vice versa but not both A and B.

The ability to specify choices via the additive connectives is also a particularly useful feature of linear logic. If we consider formulas on the left hand side of $\vdash$ as what are provided (program formulas), then A $\&$ B (*additive conjunction*) stands for one’s own choice, either of A or B but not both. A $\oplus$ B (*additive disjunction*) stands for the possibility of either A or B, but we don’t know which. In other words, while $\&$ refers to inner determinism, $\oplus$ refers to inner non-determinism. Hence, $\&$ can be used to model an agent’s own choices (*internal choices*) whereas $\oplus$ can be used to model *indeterminate possibilities* (or external choices) in the environment. The duality between $\&$ and $\oplus$, being respectively an internal and an external choice, is a well-known feature of linear logic [5].

Due to the duality between formulas on two sides of $\vdash$, formulas on the right side can be regarded as goal formulas, i.e. what to be derived. A goal $A \& B$ means that after deriving this goal, one can choose between A or B. In order to have this ability to choose, one must prepare for both cases - being able to derive A and derive B. On the other hand, a goal $A \oplus B$ means that it is not determined which goal between A and B. Hence, one can choose to derive either of them. In terms of deriving goals, $\&$ and $\oplus$ among goal formulas act as introducing indeterminate possibilities and introducing an internal choice respectively.

The temporal operators used are $\bigcirc$ (next), $\Box$ (anytime), and $\Diamond$ (sometime) [3]. Formulas with no temporal operators can be considered as being available only at present. Adding $\bigcirc$ to a formula A, i.e. $\bigcirc$A, means that A can be used only at the next time point and exactly once. Similarly, $\Box$A means that A can be used at any time (exactly once, since it is linear). $\Diamond A$ means that A can be at some time (also exactly once). Whilst the temporal operators have their standard meanings, the notions of internal and external choice can be applied here as well, in that in that $\Box A$ means that A can be used at any time (but exactly once) with the choice of time being internal to the agent, and $\Diamond A$ means that A can be used at some time with the choice of time being external to the agent.

The semantics of TLL connectives and operators as above are given via its sequent calculus, since we take a proof-theoretic approach in modeling agent interaction.

2.2 A Model for Agent Interaction

In [8], an interaction modeling framework which uses TLL as a means of specifying interaction protocols is used as TLL is natural to model resources, internal choices and indeterminate possibilities with respect to time. Various concepts such as resource, capability and commitment/goal are encoded in TLL. The symmetry between a formula and its negation in TLL is explored as a way to model resources and commitments/goals. In particular, formulas to be located on the left hand side of $\vdash$ can be regarded as formulas in supply (resources) while formulas to be located on the right hand side of $\vdash$ as formulas in demand (goals).

A unit of consumable *resources* is then modeled as a proposition in linear logic and can be preceded by temporal operators to address time dependency. For example, listening to music after (exactly) three time points is denoted as $\bigcirc \bigcirc \bigcirc music$. A shorthand is $\bigcirc^3 music$.

The *capabilities* of agents refer to producing, consuming, relocating and changing ownership of resources. Capabilities are represented by describing the state before and after performing them. The general representation form is $\Gamma \multimap \Delta$, in which Γ describes the conditions before and Δ describes the conditions after. The linear implication $\multimap$ ensures that the conditions before will be transformed into the conditions after.

To take an example, consider a capability of producing music using music player to play music files. There are two options available at the agent's own choice, one is using mp3 player to play mp3 files, the other is using CD player to play CD files. The encoding is:

$$\Box[[(mp3 \otimes mp3\_player) \oplus (CD \otimes CD\_player)] \multimap music]^1$$

where $\Box$ means that the capability can be applied at any time, $\oplus$ indicates an internal choice (not $\&$, as it is located on the left hand side of $\multimap$).

3 Desiderata for a Choice Calculus

Unpredictable changes in the environment can be regarded as a set of possibilities which the agents do not know the outcomes. There are several strategies for dealing with unpredictable changes. A safe approach is to prepare for all the possible scenarios, at the cost of extra reservation and/or consumption of resources. Other approaches are more risky in which agents make a closest possible prediction of which possibilities to occur and act accordingly. If the predictions are correct, agents achieve the goals with resource efficiency. Here, there is a trade-off between resource efficiency and safety.

[1] In the modeling, formulas representing media players are consumed away, which does not reflect the persistence of physical objects. However, we focus on modeling how resources are utilized, not their physical existences and hence simplify the encoding since it is not necessary to have the media players retained for later use.

In contrast to indeterminate possibilities, internal choices are what agents can decide by themselves to their own advantage. Decisions on internal choices can be based on what is best for the agents' current and local needs. However, it is desirable that they consider internal choices in the context of other internal choices that have been or will be made. This requires an ability to make an informed decision on internal choices. If we put information for decision making on internal choices as constraints associated with those internal choices then what required is a modeling of internal choices with their associated constraints such that agents can reason about them and decide accordingly. Also, in such a distributed environment as multi-agent systems, such a modeling should take into account as constraints the dependencies among internal choices.

In addition, as agents act in time, decisions can be made precisely at the required time or can be well prepared in advance. When to decide and act on internal choices should be at the agents' autonomy. The advantages of deciding internal choices in advance can be seen in an example as resolving a goal of $\bigcirc^3(A \oplus B)$. This goal involves an internal choice ($\oplus$) to be determined at the third next time point ($\bigcirc^3$). If the agent decides now to choose A and commits to making the same decision at the third next time point, then from now, the agent only has to focus a goal of $\bigcirc^3 A$. This also means that resources used for other goals can be guaranteed to be exclusive from the requirements of $\bigcirc^3(A \oplus B)$, which might not be the case otherwise when $\bigcirc^3(A \oplus B)$ is decided as $\bigcirc^3 B$ at the third next time point.

The following example illustrates various desirable strategies of agents.

Peter intends to organize an outdoor party in two days' time. He has a goal of providing music at the party. He has a CD burner and a blank CD onto which he can burn music in CD or mp3 format. His friend, John, can help by bringing a CD player or an mp3 player to the party but Peter will not know which until tomorrow. David then informs Peter that he would like to borrow Peter's CD burner today.

In this situation, to Peter, there is an internal choice on the music format and an indeterminate possibility regarding the player. We consider two strategies. If Peter does not let David borrow the CD burner, he can wait until tomorrow to find out what kind of player John will bring to the party and choose the music format accordingly at that time. Otherwise, he can not delay burning the CD until tomorrow and so has to make a prediction on which player John will bring to the party and then decide the internal choice on the music format early (now), burn the CD and let David borrow the CD burner. The question is then how to make such strategies available for Peter to explore.

One solution is using formalisms such as logic to enable agent reasoning on those internal choices and indeterminate possibilities. Linear Logic is highly suitable here, because it allows us to distinguish between internal determinism and non-determinism. *Temporal* linear logic (TLL) further puts such modeling of them in a time dependent context. Indeed, internal choices and external choices (inner non-determinism) have been modeled previously using Linear Logic [6, 9] and TLL [8, 2].

An important observation is that although (temporal) linear logic captures the notions of internal choice and indeterminate possibility, its sequent rules constrain agents to specific strategies and make each decision on internal choices in isolation (subject only to local information). Specifically, consider the following rules of standard sequent calculus:

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \oplus B \vdash \Delta} \quad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \,\&\, B, \Delta}$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, A \,\&\, B \vdash \Delta} \quad \frac{\Gamma, B \vdash \Delta}{\Gamma, A \,\&\, B \vdash \Delta} \quad \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \oplus B, \Delta} \quad \frac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \oplus B, \Delta}$$

where for the formulas on the left hand side of $\vdash$, $\&$ remarks internal choice and $\oplus$ remarks indeterminate possibility and vice versa for the formulas on the right hand side of $\vdash$.

The first set of rules require agents to prepare for both outcomes of the indeterminate possibility. Though this strategy is safe, it demands extra and unnecessary resources and actions. Moreover, this strategy does not take into account an agents' prediction of the environment or whether it is willing to take risks.

More importantly, according to the last set of rules, the free (internal) choice agents have is determined *locally*, i.e. without a global awareness. Hence decisions on these free choices may not be optimal. In particular, if the formula $A \& B$ (on the left hand side of $\vdash$) is not used in any proof now, without this kind of local information, the decision on this internal choice becomes unguided. Hence, if there is further information about possible future goals or about dependencies on other internal choices, this information should be considered and the agent should be enabled to decide the internal choice accordingly. Moreover, the rule does not allow agents to explore the strategy of deciding internal choices in advance.

Referring to our running example, in the first strategy, Peter does not let David borrow the CD burner, and so Peter can then find a proof using standard sequent rules to achieve the goal of providing music at the party two days later. However, in the second strategy, the search using standard TLL sequent calculus for a proof of the goal fails as it requires to have music in both formats (mp3 and CD) so as to be matched with the future possibility of the media player.

Hence, in this paper, we investigate how TLL not only allows us to model the difference between internal choice and indeterminate possibility with respect to time, but also allows us to capture *dependencies* among internal choices, constraints on how internal choices can be made as well as predictions and decisions of indeterminate possibilities. Such constraints may also reflect global consideration of other goals and other threads of interaction. We further consider strategies that can be used to deal with internal choices with respect to time, reflecting how cautious the agents are and whether the agents deal with them in advance. However, we will not discuss how agents can predict the environment outcomes correctly.

4 A Choice Calculus

If we assume that the order of operants is unchanged throughout the process of formulas manipulation, in other words, ignoring the commutative property of $\oplus$ and $\&$, then the decision on choices and indeterminate possibilities can be regarded as selecting the left hand side or the right hand side of the connective. For simplicity, we shall refer to both internal choices (choices with inner determinism) and indeterminate possibilities (choices with non-determinism) simply as choices.

As the decision on a choice is revealed at the time point associated with the choice, before that time point, the decision of the choice is unknown. We encode the decisions on choices using TLL constants.

We need to consider the base values for choice decisions, how to specify choices of the same decisions, choices that are dependent on other choices and also how standard sequent calculus rules are modified to reflect decisions on choices.

We use the notation $\overset{\&_x}{\hookrightarrow}$ or $\overset{\oplus_x}{\hookrightarrow}$ to indicate the result of the decision making of $\&_x$ and $\oplus_x$ respectively. The subscript indicates the ID of the connective. The base values for their decision can be encoded by TLL constants L, and R. For example, the result of the decision on the choice in $A \& B$ is L if A results from $A \& B$ and is R if B results. For internal choices, their decisions can be regarded as variables as agents can decide the assignment of values. Formally, we write $\vdash \overset{\&_1}{\hookrightarrow} \multimap L$ or $\overset{\&_1}{\hookrightarrow} \vdash L$ to denote that the left subformula of $\&_1$ was selected.

Decisions on indeterminate possibilities could also be represented as variables. However, we will explicitly represent that agents can not decide the outcomes of indeterminate possibilities and that the decisions on them (by external factors) have not been made by using the form $L \oplus R$ (or $L \& R$). For example, given the indeterminate possibility $\bigcirc^n(A \oplus_x B)$, we represent their decision by $\bigcirc^n(L \oplus_x R)$, where n is the time point associated with the choice and $\oplus_x$ is the same connective as that in $\bigcirc^n(A \oplus_x B)$.

By modeling the choices explicitly, we can state constraints between them. For example, if two choices, $\overset{\&_x}{\hookrightarrow}$ and $\overset{\&_y}{\hookrightarrow}$, need to be made consistently — either both right or both left — then this can be stated as $\overset{\&_x}{\hookrightarrow} = \overset{\&_y}{\hookrightarrow}$ or, in logic encoding, $\overset{\&_x}{\hookrightarrow} \vdash \overset{\&_y}{\hookrightarrow}, \overset{\&_x}{\hookrightarrow} \dashv \overset{\&_y}{\hookrightarrow}$. More generally, we can state that a given choice $\&_x$ should depend on a combination of other choices or some external constraints. We use $condL_x$ (respectively $condR_x$) to denote the condition that should hold for the left (respectively right) side of the choice to be taken. Clearly, $condL_x$ and $condR_x$ should always be mutually exclusive. These conditions, in their presence, completely determine the results of the choices' decisions. In their absence, the internal choices become truly free choices and we are getting back to the normal case as of standard sequent rules. These conditions are encoded as TLL sequents so that sub-conditions (sequents) can be found via proof search.

Given a formula Γ which contains a sub-formula A, we can compress the sequence of decisions that need to be made in order to obtain A from Γ into a single *representative choice*. For example, if $\Gamma = B \&_1 \bigcirc^a(\bigcirc^b(A \oplus_2 C) \&_3 D)$ then, in order to obtain A from Γ we need to decide on the right side of $\&_1$, then, a time units later, decide on the left side of $\&_3$, and b time units after that, have the left side of $\oplus_2$ be selected by the environment (an indeterminate possibility). Formally, the notion of representative is defined as below.

Definition 1. *A* ***representative*** *choice $\&_r$ with respect to a formula A in a compound program formula (respectively goal formula) Γ is a choice $\bigcirc^x A \&_r \bigcirc^y 1$ (respectively $\bigcirc^x A \oplus_r \bigcirc^y 1$) whose decision is L if A is chosen from Γ and is R otherwise, where $x, x \geq 0$ is the time associated with A in Γ and $y, y \geq 0$ is is the time point associated with* 1.

Note that at the time of representing the choice $\&_r$ (or $\oplus_r$), the value of y is not known. It will be known after all the decisions of internal choices and indeterminate possibilities in Γ are revealed.

In the previous example, such a sequence of decisions on internal choices and indeterminate possibilities on Γ to obtain A can be captured by the sequent:
$\vdash (\overset{\&_1}{\hookrightarrow}\!\multimap R) \otimes \bigcirc^a(\overset{\&_3}{\hookrightarrow}\!\multimap L) \otimes \bigcirc^{a+b}(\overset{\oplus_2}{\hookrightarrow}\!\multimap L)$.
This is the determining condition for A to be obtained from Γ. Observe that we can compress Γ into a representative choice for A of the form $\bigcirc^{a+b}A \mathbin{\&_r} \bigcirc^y)$ such that the choice $\&_r$ is decided left if $\bigcirc^{a+b}A$ results from F and is decided right otherwise. The condition above then corresponds to $condL_r$ of $\&_r$. As being mutually exclusive, $condR_r$ is captured as:
$\vdash (\overset{\&_1}{\hookrightarrow}\!\multimap L) \oplus \bigcirc^a(\overset{\&_3}{\hookrightarrow}\!\multimap R) \oplus \bigcirc^{a+b}(\overset{\oplus_2}{\hookrightarrow}\!\multimap R)$.

We now come to determine sequent calculus rules for various strategies on choices.

4.1 Extended Sequent Calculus

We take a general assumption that regarding indeterminate possibilities, the environment (or external factors) determines the outcomes of the possibilities at their associated times. For example, given the indeterminate possibility $\bigcirc^4(A \oplus B)$, after four time points, the environment determines the possibility such that $\bigcirc^4(A \oplus B)$ becomes $\bigcirc^4 A$ or $\bigcirc^4 B$ and only at this time, the outcome becomes known to agents. This assumption is based on the inherent modeling of TLL that formulas denoted at a specific time point last only in that time point.

The standard sequent calculus rules for indeterminate possibilities, which demand that agents prepare for all possibilities, correspond to a safe approach. However, if the agent chooses a strategy of risk taking, it then makes predictions of the outcome of the indeterminate possibility and follows the search path corresponding to the predicted one. The sequent rules for such a strategy are ($\vdash_{cc}$ means $\vdash$ in choice calculus context):

$$\frac{\Gamma \vdash_{cc} F, \Delta \quad [L \vdash L \mathbin{\&_n} R]}{\Gamma \vdash_{cc} F \mathbin{\&_n} G, \Delta} \qquad \frac{\Gamma \vdash_{cc} G, \Delta \quad [R \vdash L \mathbin{\&_n} R]}{\Gamma \vdash_{cc} F \mathbin{\&_n} G, \Delta}$$

$$\frac{\Gamma, F \vdash_{cc} \Delta \quad [L \oplus_n R \vdash L]}{\Gamma, F \oplus_n G \vdash_{cc} \Delta} \qquad \frac{\Gamma, G \vdash_{cc} \Delta \quad [L \oplus_n R \vdash R]}{\Gamma, F \oplus_n G \vdash_{cc} \Delta}$$

where formulas in square brackets are the conditions (predictions) and those outside brackets are the main formulas. The conditions are evaluated independently from the main formulas and at the time associated with the indeterminate possibility, when the environment reveals the outcomes. If there is a proof of the main formulas, and if the conditions are also satisfied, then the proof search is successful. If the conditions can not be satisfied even though there is a proof among the main formulas, then the search for proof fails on this branch associated with the conditions.

Moreover, if the agent further decides upon its prediction of an indeterminate possibility before the time associated with the possibility, it also can bring out the possibility's outcome earlier in the search:

$$\frac{\Gamma \vdash_{cc} \bigcirc^x F, \Delta \quad [L \vdash L \mathbin{\&_n} R]}{\Gamma \vdash_{cc} \bigcirc^x(F \mathbin{\&_n} G), \Delta} \qquad \frac{\Gamma \vdash_{cc} \bigcirc^x G, \Delta \quad [R \vdash L \mathbin{\&_n} R]}{\Gamma \vdash_{cc} \bigcirc^x(F \mathbin{\&_n} G), \Delta}$$

$$\frac{\Gamma, \bigcirc^x F \vdash_{cc} \Delta \quad [L \oplus_n R \vdash L]}{\Gamma, \bigcirc^x (F \oplus_n G) \vdash_{cc} \Delta} \qquad \frac{\Gamma, \bigcirc^x G \vdash_{cc} \Delta \quad [L \oplus_n R \vdash R]}{\Gamma, \bigcirc^x (F \oplus_n G) \vdash_{cc} \Delta}$$

Internal choices are decided by the owner agent at the time associated with the choice, subject to any constraints ($condL_n$ or $condR_n$) imposed on them. The following sequent rules reflect that:

$$\frac{\Gamma, F \vdash_{cc} \Delta \quad (\vdash condL_n)}{\Gamma, F \&_n G \vdash_{cc} \Delta} \qquad \frac{\Gamma, G \vdash_{cc} \Delta \quad (\vdash condR_n)}{\Gamma, F \&_n G \vdash_{cc} \Delta}$$

$$\frac{\Gamma \vdash_{cc} F, \Delta \quad (\vdash condL_n)}{\Gamma \vdash_{cc} F \oplus_n G, \Delta} \qquad \frac{\Gamma \vdash_{cc} G, \Delta \quad (\vdash condR_n)}{\Gamma \vdash_{cc} F \oplus_n G, \Delta}$$

where $condL_n$ and $condR_n$ are conditions imposed on the internal choice n for the choice to be decided left or right. These conditions may or may not be present.

Moreover, if the agent is to decide the choice priorly, it can bring out the choice's outcome earlier in the search:

$$\frac{\Gamma, \bigcirc^x F \vdash_{cc} \Delta \quad (\vdash condL_n)}{\Gamma, \bigcirc^x (F \&_n G) \vdash_{cc} \Delta} \qquad \frac{\Gamma, \bigcirc^x G \vdash_{cc} \bigcirc^x \Delta \quad (\vdash condR_n)}{\Gamma, \bigcirc^x (F \&_n G) \vdash_{cc} \bigcirc^x \Delta}$$

$$\frac{\Gamma \vdash_{cc} \bigcirc^x F, \Delta \quad (\vdash condL_n)}{\Gamma \vdash_{cc} \bigcirc^x (F \oplus_n G), \Delta} \qquad \frac{\Gamma \vdash_{cc} G, \bigcirc^x \Delta \quad (\vdash condR_n)}{\Gamma \vdash_{cc} \bigcirc^x (F \oplus_n G), \Delta}$$

These above sequent rules, together with standard TLL sequent rules, form the *choice calculus*.

Considering our running example, recall that if Peter is to let David borrow the CD burner now, he needs to decide on the music format (the internal choice $\&_1$) now. This involves making a prediction on the player that John will possibly bring. For instance, Peter predicts that John will provide an mp3 player (i.e. $L \oplus_3 R \vdash L$). Using the choice calculus, this is captured by the following inference:

$$\frac{\Gamma, \bigcirc^2 mp3\_player \vdash_{cc} \bigcirc^2 music \quad [L \oplus_3 R \vdash L]}{\Gamma, \bigcirc(\bigcirc mp3\_player \oplus_3 \bigcirc CD\_player) \vdash_{cc} \bigcirc^2 music}$$

Based on this prediction, agent Peter decides early on the choice of music format $\&_1$ (mp3 format now) and burns the blank CD accordingly. By taking this risk on the prediction, agent Peter then successfully obtains a proof of $\bigcirc^2 music$ (given below). If the prediction $L \oplus_3 R \vdash L$ is provable at the next two days, then the goal is achieved.

For the purposes of presenting the proof we make the following abbreviations.

Let B (for "Burn") denote the formula

$$\Box[Blank\_CD \otimes CD\_Burner \multimap CD\_Burner \otimes (\Box mp3 \&_1 \Box CD)]$$

i.e. one can convert a blank CD to either an mp3 or music format CD (internal choice of which).

Let P (for "Play") denote the formula

$$\Box[[(mp3 \otimes mp3\_player) \oplus_2 (CD \otimes CD\_player)] \multimap music]$$

i.e. at any time, either using mp3 player on mp3 music or CD player on a CD, one can produce music (the choice $\oplus_2$ here is internal).

Let R (for "Resources") denote the formula

$$\Box Blank\_CD \otimes \Box CD\_Burner.$$

Let J (for "John", i.e. the music player that John will provide) denote the formula

$$\bigcirc[\bigcirc mp3\_player \oplus_3 \bigcirc CD\_player]$$

i.e. either an mp3 player or CD player will be provided after two days. $\oplus_3$ is an indeterminate possibility to Peter and will be revealed tomorrow.

We also abbreviate $music$ to m, and $player$ to p, e.g. $mp3\_player$ becomes $mp3p$, then we have the following proof of $\bigcirc^2 music$ where some inferences combine a number of rule applications, and where (for space reasons) we have left out giving the CD burner to David at the rule marked "$\otimes, \Box, \multimap$". As there is no imposed condition for $\&_1$ ($condL_1 = 1$), it is omitted in the proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{mp3 \vdash mp3} \quad \dfrac{}{mp3p \vdash mp3p}}{mp3, mp3p \vdash mp3 \otimes mp3p}\otimes}{mp3, mp3p \vdash (mp3 \otimes mp3p) \oplus_2 (cd \otimes cdp)}\oplus_2}{\Box mp3, P, \bigcirc^2 mp3p \vdash \bigcirc^2 m}\bigcirc^2, \Box, \multimap}{\Box mp3 \,\&_1\, \Box cd, P, \bigcirc^2 mp3p \vdash \bigcirc^2 m}\&_1}{R, P, B, \bigcirc^2 mp3p \vdash \bigcirc^2 m}\otimes, \Box, \multimap \qquad [L \oplus_3 R \vdash L]}{R, F, P, B \vdash \bigcirc^2 m}\bigcirc\oplus_3$$

In this example we begin (bottom-most inference) by making an "in-advance" decision of the choice $\oplus_3$, specifically we predict that John will provide an MP3 player. We then use standard TLL sequent rules to burn an MP3 format CD. When the time comes to make a decision for $\oplus_2$ we can select to use the MP3 player to produce music. As can be seen from the example, internal choices and indeterminate possibilities are properly modeled with respect to time. Moreover, several strategies are enabled at agent Peter due to the use of choice calculus. If Peter is to take a safe approach, he should delay deciding the music format until tomorrow and ignores David's request. If Peter is willing to take risks, he can predict the indeterminate possibility of which player John will bring to the party and act accordingly. Peter can also decide the choice on music early so as to lend David the CD burner.

Hence, these sequent calculus rules are in place to equip agents with various strategies for reasoning to deal with indeterminate possibilities and internal choices. These strategies make it more flexible to deal with changes and handle exceptions with global awareness and dependencies among choices. In the next section, we explore an application of such modeling of choices and their coping strategies, especially dependencies among choices, to distributed problem solving in a flexible interaction modeling TLL framework [8]. But first, we show that proofs using the additional rules are, in a sense, equivalent to proofs in the original TLL sequent calculus.

The intuition behind the soundness and completeness properties of proofs using these additional rules with respect to proofs which only use original TLL sequent calculus is that eventually indeterminate possibilities like between A and B will be revealed as the outcome turns out to be one of the two. The soundness and completeness properties are then evaluated and proved in this context. In particular, we introduce the concept of a revealed proof, which is a proof in which all the internal choices and possibilities are revealed and replaced by the actual respective outcomes. As a result of such replacements, all of the additional rules added in our choice calculus collapse to sequents, leaving only the standard TLL rules. Note that the proofs using choice calculus require that all the assumptions will turn out to be correct. Clearly, if the assumptions turn out to be unfounded, then the proofs are not valid.

Definition 2. *The* ***revealed proof*** *corresponding to a given proof of $\Gamma \vdash \Delta$ is the proof resulting from replacing all occurrences of choices with the actual outcomes of these choices. That is, any formula $F \oplus G$ corresponding to an indeterminate possibility is replaced by either F or G, corresponding to the decision that was made by the environment; and any formula $F \,\&\, G$ corresponding to an internal choice is replaced by either F or G, corresponding to the choice that was made by the agent.*

Theorem 1 (Soundness).

A revealed proof of a proof using the TLL sequent rules augmented with the additional choice calculus rules is a valid proof under standard TLL sequent calculus rules. ***Proof sketch:*** *All of the additional rules introduced by the choice calculus disappear when the proof is made into a revealed proof. For example, consider the rules (on the left) which are replaced in a revealed proof, where $F \,\&\, G$ is replaced by F, by the identities on the right.*

$$\frac{\Gamma \vdash_{cc} F, \Delta \quad [L \vdash L \,\&_n\, R]}{\Gamma \vdash_{cc} F \,\&_n\, G, \Delta} \qquad \frac{\Gamma \vdash F, \Delta}{\Gamma \vdash F, \Delta}$$

$$\frac{\Gamma \vdash_{cc} \bigcirc^x F, \Delta \quad [L \vdash L \,\&_n\, R]}{\Gamma \vdash_{cc} \bigcirc^x (F \,\&_n\, G), \Delta} \qquad \frac{\Gamma \vdash \bigcirc^x F, \Delta}{\Gamma \vdash \bigcirc^x F, \Delta}$$

$$\frac{\Gamma, F \vdash_{cc} \Delta}{\Gamma, F \,\&_n\, G \vdash_{cc} \Delta} \qquad \frac{\Gamma, F \vdash \Delta}{\Gamma, F \vdash \Delta}$$

As a result of this theorem, it then becomes that a proof under choice calculus is sound if the assumptions (predictions) it relies on are correct.

Moreover, as choice calculus also contains standard TLL sequent calculus rules, the completeness property holds trivially.

Theorem 2 (Completeness). *A proof using standard TLL sequent calculus rules is also a proof under choice calculus.*

5 Splitting a Formula

Interaction between agents is often necessary for the achievement of their goals. In the above example with Peter and John, if Peter had a CD player of his own, he would not need to interact with John in order to have music at the party. In general, it will be necessary for an agent to co-ordinate interaction with many different agents, the precise number and identity of which may not be known in advance. In order to achieve this, in this section we investigate a mechanism for partial achievement of a goal. In particular, this is a process of decomposing a given TLL goal formula into concurrent subgoals.

For example, assume that Peter now has the additional goal of having either Chinese or Thai food at the party. Deriving which goal - Chinese food (abbreviated as C) or Thai food (abbreviated as T) - is an internal choice ($\oplus_3$). Peter's goal is then

$$CD\_Burner \otimes \bigcirc^2[music \otimes (C \oplus_3 T)]$$

However, Peter can not provide food, but his friends, Ming and Chaeng, can make Chinese food and Thai food respectively. Hence, this goal can not be fulfilled by Peter alone but involves interaction with John and David as above and also Ming or Chaeng. If this goal is sent as a request to any one of them, none would be able to fulfill the goal in its entirety. Hence, it is important that the goal can be split up and achieved partially via concurrent threads of interaction. In this case, we would split this into the sub-goal $CD\_Burner \otimes \bigcirc^2 music$, which is processed as above, the sub-goal $\bigcirc^2 C \oplus_4 \bigcirc^2 1$, which is sent as a request to Ming, and the sub-goal $\bigcirc^2 1 \oplus_4 \bigcirc^2 T$, which is sent as a request to Chaeng. The choice $\oplus_4$ will be later determined consistently with $\oplus_3$.

Hence we need to be able to split a goal into sub-goals, and to keep track of which parts have been achieved. In particular, it is useful to isolate a sub-goal from the rest of the goal. We do this by taking the overall formula Γ and separating from it a particular sub-formula A. We show how this can be done on the fragment which contains the connectives $\otimes, \oplus, \&, \bigcirc$.

The split-ups of a formula Γ with respect to the formula A that Γ contains are the two formulas $\widehat{\Gamma - A}$ and $\widehat{A}$, which are defined below.

$\widehat{\Gamma - A}$ is the formula Γ which has undergone a single removal or substitution of (one occurrence of) A by 1 while the rest is kept unchanged. Specifically, where A resides in the structure of Γ, the following mapping is applied to A and its directly connected formulas Δ. Δ is any TLL formula and $x \geq 0$.

1. $A \mapsto 1$
2. $\bigcirc^x A \mapsto \bigcirc^x 1$
3. $\bigcirc^x A \; op \; \Delta \mapsto \bigcirc^x 1 \; op \; \Delta$ for $op \in \{\otimes, \&, \oplus\}$

We also apply the equivalence $1 \otimes \Delta \equiv \Delta$, so that $\bigcirc^x A \otimes \Delta \mapsto \Delta$.

The formula $\widehat{A}$ is determined recursively according to structure of Γ as below, by examining the structure of Γ:

- If $\Gamma^1 = \bigcirc^x A$, then $\widehat{A^1} = \bigcirc^x A$
- If $\Gamma^1 = \bigcirc^x A \; op_m \; \Delta$, then $\widehat{A^1} = \bigcirc^x A \; op_m \; 1$
- If $\Gamma^n = \bigcirc^x \Gamma^{n-1}$, then $\widehat{A^n} = \bigcirc^x \widehat{A^{(n-1)}}$
- If $\Gamma^n = \Gamma^{n-1} \; op_n \; \Delta$, then $\widehat{A^n} = \widehat{A^{n-1}} \; op_n \; 1$

where Γ^i, Δ are formulas of the fragment and Γ^i contains A. $op_n, op_m \in \{\otimes, \&, \oplus\}$ and n, m are the IDs. We also again apply the equivalence $1 \otimes \Delta \equiv \Delta$, so that when $\Gamma^1 = \bigcirc^x A \otimes \Delta$, then $\widehat{A^1} = \bigcirc^x A$.

Another view is that $\widehat{A}$ is obtained by recursively replacing formulas that rest on the other side of connective (to the formula that contains A) by 1 if the connective is $\oplus$ or $\&$ and remove them if the connective is $\otimes$.

It can be seen from the formulation of $\widehat{\Gamma - A}$ and $\widehat{A}$ that there are requirements of choice dependencies among the split ups. Indeed, all the corresponding choices and possibilities in them must be consistent. In particular, decisions made on the corresponding choices and possibilities in $\widehat{\Gamma - A}$, and $\widehat{A}$ should be the same as those that would have been made on the corresponding ones in Γ. Indeed, if A is ever resulted from Γ as a result of a sequence of choices and possibilities in Γ being decided, then those decisions also make $\widehat{A}$ become A.

As an example, we return to our running example and consider Peter's goal formula. The goal $G = CD\_Burner \otimes \bigcirc^2[music \otimes (C \oplus_3 T)]$ can be split into:
$[\widehat{G - C}] = CD\_Burner \otimes \bigcirc^2[music \otimes (1 \oplus_3 T)]$ and $\widehat{C} = \bigcirc^2(C \oplus_3 1)$.
Subsequently, $\widehat{G - C}$ can be split into:
$[G \widehat{- C -} T] = CD\_Burner \otimes \bigcirc^2 music$ and $\widehat{T}$ of $\widehat{G - C}$ is $\bigcirc^2(1 \oplus_3 T)$.

Indeed, $\widehat{A}$ can result in $\bigcirc^x A$ or $\bigcirc^y 1$, $x, y \geq 0$, as a result of having all the choices in $\widehat{A}$ decided. In the following theorem, we show that $\widehat{A}$ can be compressed into a representative choice (of A in $\widehat{A}$) of the form $\bigcirc^x A \mathbin{\&}_r \bigcirc^y 1$ if $\widehat{A}$ is a program formula, or $\bigcirc^x A \oplus_r \bigcirc^y 1$ if $\widehat{A}$ is a goal formula.

Theorem 3. *$\widehat{A} \vdash_{cc} \bigcirc^x A \mathbin{\&}_r \bigcirc^y 1$ if $\widehat{A}$ is a program formula, and $\widehat{A} \vdash_{cc} \bigcirc^x (A \oplus_r 1)$ if $\widehat{A}$ is a goal formula, where $x, x \geq 0$ is the time associated with the occurrence of A in $\widehat{A}$ and for some value of $y \geq 0$. Additionally, $\bigcirc^x A \mathbin{\&}_r \bigcirc^y 1 \vdash_{cc} \widehat{A}$ and $\bigcirc^x (A \oplus_r 1) \vdash_{cc} \widehat{A}$ (proof omitted).*
Proof*: by induction on the structure of $\widehat{A}$. We highlight a few cases of the proof for $\widehat{A} \vdash_{cc} \bigcirc^x A \mathbin{\&}_r \bigcirc^y 1$. The others are similar.*

Base step*: $\widehat{A} = A$, hence $x = 0, condL_r = 1$. The choice is decided left and we have $A \vdash_{cc} A$.*

Induction step*: Assume the hypothesis is true for n, so that $\widehat{A^n} \vdash_{cc} \bigcirc^n A \mathbin{\&}_n \bigcirc^y 1$ is provable, which means the following (upper) sequents are also provable:*

$$\frac{\widehat{A^n} \vdash_{cc} \bigcirc^n A \; [\vdash condL_n]}{\widehat{A^n} \vdash_{cc} \bigcirc^n A \mathbin{\&}_n \bigcirc^y 1} \mathbin{\&}_n \qquad \frac{\widehat{A^n} \vdash_{cc} \bigcirc^y 1 \; [\vdash condR_n]}{\widehat{A^n} \vdash_{cc} \bigcirc^n A \mathbin{\&}_n \bigcirc^y 1} \mathbin{\&}_n$$

We show the case for $\widehat{A^{n+1}} = \widehat{A^n} \mathbin{\&}_1 1$ below, the others $\bigcirc \widehat{A^n}$, and $\widehat{A^n} \oplus_2 1$ are similar. In this case, we need to prove $\widehat{A^n} \mathbin{\&}_1 1 \vdash_{cc} \bigcirc^n A \mathbin{\&}_{n+1} \bigcirc^y 1$, where $condL_{n+1} = condL_n \otimes (\overset{\&_1}{\hookrightarrow}\!\multimap L)$; and $condR_{n+1} = condR_n \oplus (\overset{\&_1}{\hookrightarrow}\!\multimap R)$

$$\frac{\dfrac{\widehat{A^n} \vdash_{cc} \bigcirc^n A \; [\vdash condL_n]}{\widehat{A^n} \mathbin{\&}_1 1 \vdash_{cc} \bigcirc^n A \; [\vdash condL_n \otimes (\overset{\&_1}{\hookrightarrow}\!\multimap L)]} \mathbin{\&}_1}{\widehat{A^n} \mathbin{\&}_1 1 \vdash_{cc} \bigcirc^n A \mathbin{\&}_{n+1} \bigcirc^y 1} \mathbin{\&}_{n+1} \qquad \frac{\dfrac{\widehat{A^n} \vdash_{cc} \bigcirc^y 1 \; [\vdash condR_n] \quad \dfrac{1 \vdash_{cc} 1 (y=0)}{1 \vdash_{cc} \bigcirc^y 1}}{\widehat{A^n} \mathbin{\&}_1 1 \vdash_{cc} \bigcirc^y 1 \; [\vdash condR_n \oplus (\overset{\&_1}{\hookrightarrow}\!\multimap R)]} \mathbin{\&}_1}{\widehat{A^n} \mathbin{\&}_1 1 \vdash_{cc} \bigcirc^n A \mathbin{\&}_{n+1} \bigcirc^y 1} \mathbin{\&}_{n+1}$$

where the value of y is assigned as appropriately in the proof. Both cases of the decision on $\mathbin{\&}_1$ are proved.

Applying this theorem to the above example, we can obtain further results:
$\widehat{C} = \bigcirc^2(C \oplus_3 1) = \bigcirc^2 C \oplus_4 \bigcirc^2 1$,
$\widehat{T}$ (of $\widehat{G - C}$) $= \bigcirc^2(1 \oplus_3 T) = \bigcirc^2 1 \oplus_4 \bigcirc^2 T$, where $\oplus_4$ is the representative choice and is of the same decision as $\oplus_3$ at the next two time points.

The equivalence relationship between Γ and its split ups, $\widehat{\Gamma - A}$ and $\widehat{A}$, is established by the following theorems.

Theorem 4. $\widehat{A}, \widehat{\Gamma - A} \vdash_{cc} \Gamma$.
(From the multiplicative conjunction of the split ups of Γ via A — $\widehat{A}, \widehat{\Gamma - A}$ — we can derive Γ).
Proof (sketch)*: by induction on the structure of Γ. We highlight a few cases of the proof. The others are similar.*

Base step*:*
Case *$\Gamma = A \oplus_1 \Delta$. We need to prove $A \oplus_1 1, 1 \oplus_1 \Delta \vdash_{cc} A \oplus_1 \Delta$. Both choices for $\oplus_1$ fulfill this, as below.*

$$\dfrac{\dfrac{A \vdash_{cc} A}{A, 1 \vdash_{cc} A}}{A \oplus_1 1, 1 \oplus_1 \Delta \vdash_{cc} A \oplus_1 \Delta}\ \oplus R \qquad \dfrac{\dfrac{\Delta \vdash_{cc} \Delta}{1, \Delta \vdash_{cc} \Delta}}{A \oplus_1 1, 1 \oplus_1 \Delta \vdash_{cc} A \oplus_1 \Delta}\ \oplus R$$

Induction step*: Assume the hypothesis is true for n, so that $\widehat{A^n}, [\widehat{\Gamma - A}]^n \vdash_{cc} \Gamma^n$. We need to prove that this holds for $n+1$. We show the case for $\Gamma^{n+1} = \Gamma^n \&_1 \Delta$ below; the others ($\bigcirc^x \Gamma^n$, $\Gamma^n \otimes \Delta$ and $\Gamma^n \oplus_2 \Delta$) are all similar. In this case we have $[\widehat{\Gamma - A}]^{n+1} = [\widehat{\Gamma - A}]^n \&_1 \Delta$, and $\widehat{A^{n+1}} = \widehat{A^n} \&_1 1$.*

$$\dfrac{\dfrac{[R \vdash L \&_1 R]}{\widehat{A^n}, [\widehat{\Gamma - A}]^n \vdash_{cc} \Gamma^n\ [R \vdash L \&_1 R]}}{\widehat{A^n} \&_1 1, [\widehat{\Gamma - A}]^n \&_1 \Delta \vdash_{cc} \Gamma^n \&_1 \Delta}\ \& R \qquad \dfrac{\dfrac{\dfrac{[L \vdash L \&_1 R]}{\Delta \vdash_{cc} \Delta\ [L \vdash L \&_1 R]}}{1, \Delta \vdash_{cc} \Delta\ [L \vdash L \&_1 R]}}{\widehat{A^n} \&_1 1, [\widehat{\Gamma - A}]^n \&_1 \Delta \vdash_{cc} \Gamma^n \&_1 \Delta}\ \& R$$

Hence, both cases of the decision on $\&_1$ are proved.

One further point to note is the use of $\perp$. In our modeling context, $\perp$ does not produce any resource nor consume any other resource. We make an assumption that $\perp$ can be removed from agents' states of resources. This is formalized as a new axiom:

$$\dfrac{}{\Gamma, \perp \Vvdash_{cc} \Gamma}$$

where $\Vvdash_{cc}$ denotes $\vdash_{cc}$ under this assumption. Based on this assumption, we derive

Theorem 5. $\Gamma, \widehat{A^\perp}, \widehat{A} \Vvdash_{cc} \widehat{\Gamma - A} \otimes \widehat{A}$.
That is, from Γ and its split up on A, $\widehat{A}$, as well as $\widehat{A^\perp}$ with the same structure as of $\widehat{A}$, we can derive a multiplicative conjunction of its split ups $\widehat{A}$ and $\widehat{\Gamma - A}$.
Proof (sketch)*: by induction on the structure of Γ, where $\widehat{A^\perp}$ is obtained from $\widehat{A}$ by replacing the single copy of A by $A^\perp$. The proof can be obtained similarly from the proof of theorem 4 and is omitted here for space reason.*

Hence, $\Gamma, \widehat{A^\perp}, \widehat{A} \Vvdash_{cc} \widehat{\Gamma - A} \otimes \widehat{A} \vdash_{cc} \Gamma$.

As $\widehat{A}, \widehat{A^\perp} \vdash_{cc} \perp$, in terms of resources, the concurrent presence of both $\widehat{A}$ and its consumption $\widehat{A^\perp}$ does not consume any resource nor produce any. Hence, the presence of both does not make any effect and hence can be ignored. In terms of resources, using Γ, one can derive $\widehat{\Gamma - A} \otimes \widehat{A}$.

Theorems 4 and 5 lay important foundation of splitting up resources and goals in agent interaction. Particularly, if a goal Γ contains a formula A that the current interaction can derive, then Γ can be split into $\widehat{A}$ and $\widehat{\Gamma - A}$. If A is ever chosen in Γ, then the goal $\widehat{A}$ becomes a goal of A which can be achieved immediately by the current interaction. Similarly, if a resource Γ, which contains A, is available for use in an interaction that only uses A than the resource Γ can be split into two resources $\widehat{\Gamma - A}$ and $\widehat{A}$, of which $\widehat{A}$ can be used right away if A is ever chosen in Γ.

Returning to our example, the above theorems can be applied so that Peter can turn its goal into concurrent sub-goals $CD\_Burner \otimes \bigcirc^2 music \otimes (\bigcirc^2 C \oplus_4 \bigcirc^2 1) \otimes (\bigcirc^2 1 \oplus_4 \bigcirc^2 T)$, where the decision on $\oplus_4$ now is the same as that of $\oplus_3$ at the next two days. Therefore, agent Peter can achieve the two sub-goals $CD\_Burner \otimes \bigcirc^2 music$ as above and sends the subgoal $(\bigcirc^2 C \oplus_4 \bigcirc^2 1)$ as a request to Ming and the subgoal $(\bigcirc^2 1 \oplus_4 \bigcirc^2 T)$ as a request to Chaeng.

If Ming makes Chinese food, then $\bigcirc^2 C$ $[\overset{\oplus_4}{\hookrightarrow}\vdash L]$ is resulted. As the choice $\oplus_4$ is decided left, the other subgoal $(\bigcirc^2 1 \oplus_4 \bigcirc^2 T)$ becomes $\bigcirc^2 1$, which is also readily achievable. If Ming does not make Chinese food, there is a proof of $\bigcirc^2 1$, where $[\overset{\oplus_4}{\hookrightarrow}\vdash R]$. This decision on the choice $\oplus_4$ (choosing right) makes the subgoal $(\bigcirc^2 1 \oplus_4 \bigcirc^2 T)$ becomes $\bigcirc^2 T$. Thus, if all the subgoals are successful, this mechanism ensures that only one kind of food is made.

Hence, such splitting up of formulas allows Peter to concurrently and partially achieve its goal via different threads of interaction.

6 Discussion and Conclusion

The paper addresses issues in agents' decision making when it comes to agents' choices and indeterminate possibilities in a distributed environment. A modeling of internal choices and indeterminate possibilities as well as their decisions is presented via choice calculus. The modeling supports decisions across time, decisions based on predictions of changes in the environment, as well as dependencies and distribution among choices with respect to time.

Temporal linear logic has been used in our modeling due to its natural role in supporting agent planning in concurrent and resource-conscious agent systems. Its limitation that the standard sequent calculus rules only provide a strategy of being safe by always taking all future options into account is overcome. Indeed, our choice calculus provides agents with various strategies at each decision making point when it comes to internal choices and future possibilities. In particular, agents can make predictions of future events and/or can decide early future decisions and act accordingly. The combinations of these strategies reflect how cautious the agents are when dealing future changes, how agents strike the balance between safety and resource efficiency, how agents match up their plans with the future via predictions and how agents shape their future actions by early decisions. Moreover, as these strategies add flexibility into agents' decision making to deal with choices and changes, this is a step forward in providing flexible agent interaction.

Furthermore, the ability to deal with dependencies among distributed choices opens up another area for enhancing the quality of agents' decision making. Indeed, consideration of other or future choices or events can be specified as constraints to be satisfied on current choices. Hence, decision making by agents on choices is not carried out locally but with global and temporal awareness, and in a distributed manner.

Our second contribution is deriving a mechanism for agent reasoning to divide tasks into multiple subtasks which can be attempted concurrently in a distributed manner. In other words, rather than having human designers specify the distribution of concurrent tasks for agents, we can have agents construct a distributed model of task resolution by themselves. The mechanism is based on transferring inner dependencies into outer dependencies among distributed formulas. This is well suited to the nature of systems composed of multiple independent agents interacting with each other.

The mechanism also supports the notion of arbitrary partial achievement of goals and partial utilization of resources. This removes the need to pre-specify subgoals for various threads of interaction and lets agents work out the partial achievement of the goals and what remain. Interaction then can take place at agents' discretion, so long as it is beneficial to agents' goals. This further provides agents with an autonomy in interacting in open systems.

Our further work includes extending the choice calculus to other temporal operators like $\Box$ and $\Diamond$. We will also explore variations of the splitting up of formulas which directly encode various strategies of agents in dealing with choices. Furthermore, deriving an implementation platform using choice calculus and splitting up mechanisms for such a modeling of flexible agent interaction using TLL as [2] is also considered. Finally, there is scope for investigating the relationship between our approach for modeling choices, and the use of Computational tree logic (CTL).

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A Sequent Calculus for TLL (extract)

$$\frac{}{p \vdash p}\ axiom \qquad \frac{\Gamma \vdash A, \Delta \qquad \Gamma', A \vdash \Gamma'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ cut$$

$$\frac{\Gamma, A, B, \Gamma' \vdash \Delta}{\Gamma, B, A, \Gamma' \vdash \Delta}\ EL \quad \frac{\Gamma \vdash \Delta, A, B, \Delta'}{\Gamma \vdash \Delta, B, A, \Delta'}\ ER \quad \frac{\Gamma \vdash \Delta}{\Gamma, 1 \vdash \Delta}\ 1L \quad \frac{}{\vdash 1}\ 1R$$

$$\frac{}{\bot \vdash}\ \bot L \quad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \bot, \Delta}\ \bot R \quad \frac{}{\Gamma, 0 \vdash \Delta}\ 0L \quad \frac{}{\Gamma \vdash \top, \Delta}\ \top R$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, A^{\bot} \vdash \Delta}\ -^{\bot}L \quad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash A^{\bot}, \Delta}\ -^{\bot}R$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \otimes B \vdash \Delta}\ \otimes L \quad \frac{\Gamma \vdash A, \Delta \qquad \Gamma' \vdash B, \Delta'}{\Gamma, \Gamma' \vdash A \otimes B, \Delta, \Delta'}\ \otimes R$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, A \,\&\, B \vdash \Delta}\ \&L \quad \frac{\Gamma, B \vdash \Delta}{\Gamma, A \,\&\, B \vdash \Delta}\ \&L \quad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \,\&\, B, \Delta}\ \&R$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \oplus B \vdash \Delta}\ \oplus L \quad \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \oplus B, \Delta}\ \oplus R \quad \frac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \oplus B, \Delta}\ \oplus R$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma', B \vdash \Delta'}{\Gamma, \Gamma', A \parr B \vdash \Delta, \Delta'}\ \parr L \quad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \parr B, \Delta}\ \parr R$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma', B \vdash \Delta'}{\Gamma, \Gamma', A \multimap B \vdash \Delta, \Delta'}\ \multimap L \quad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \multimap B, \Delta}\ \multimap R$$

$$\frac{\Gamma, A[t/x] \vdash \Delta}{\Gamma, \forall x.A \vdash \Delta}\ \forall L \quad \frac{\Gamma \vdash A[y/x], \Delta}{\Gamma \vdash \forall x.A, \Delta}\ \forall R \quad \frac{\Gamma, A[y/x] \vdash \Delta}{\Gamma, \exists x.A \vdash \Delta}\ \exists L \quad \frac{\Gamma \vdash A[t/x], \Delta}{\Gamma \vdash \exists x.A, \Delta}\ \exists R$$

$$\frac{A, \Gamma \vdash \Delta}{\Box A, \Gamma \vdash \Delta}\ \Box L \qquad \frac{!\Gamma, \Box\Delta \vdash A, \Diamond\Lambda, ?\Sigma}{!\Gamma, \Box\Delta \vdash \Box A, \Diamond\Lambda, ?\Sigma}\ \Box R$$

$$\frac{!\Gamma, \Box\Delta, A \vdash \Diamond\Lambda, ?\Sigma}{!\Gamma, \Box\Delta, \Diamond A \vdash \Diamond\Lambda, ?\Sigma}\ \Diamond L \qquad \frac{\Gamma \vdash A.\Delta}{\Gamma \vdash \Diamond A, \Delta}\ \Diamond R$$

$$\frac{!\Gamma, \Box\Delta, \Xi \vdash A, \Phi, \Diamond\Lambda, ?\Pi}{!\Gamma, \Box\Delta, \bigcirc\Xi \vdash \bigcirc A, \overline{\bigcirc}\,\Phi, \Diamond\Lambda, ?\Pi}\ \bigcirc \qquad \frac{!\Gamma, \Box\Delta, \Xi, A \vdash \Phi, \Diamond\Lambda, ?\Pi}{!\Gamma, \Box\Delta, \bigcirc\Xi, \overline{\bigcirc}\,A \vdash \overline{\bigcirc}\,\Phi, \Diamond\Lambda, ?\Pi}\ \overline{\bigcirc}$$

$$\frac{!\Gamma, \Box\Delta, \Xi \vdash \Phi, \Diamond\Lambda, ?\Pi}{!\Gamma, \Box\Delta, \bigcirc\Xi \vdash \overline{\bigcirc}\,\Phi, \Diamond\Lambda, ?\Pi}\ \bigcirc \to \overline{\bigcirc}$$